CHAPTER 1 Measurement

PHYSICAL QUANTITIES

- **Physical quantities** are properties in physics that can be measured or calculated.
- **Base quantities** - physical quantities that are most fundamental and are independent of other quantities. They do not vary with time, are accessible and can be accurately reproduced

Base Quantity (symbol)	SI base Unit (symbol)
Mass (m)	kilogram (kg)
Length (*l*)	metre (m)
Time (t)	second (s)
Temperature (T)	kelvin (K)
Electric current (I)	ampere (A)
Amount of substance (n)	mole (mol)

- **Derived quantities** can be expressed in terms of products or quotients the base quantities and their SI units expressed in terms of base SI units.
- **Prefixes** can be added to SI base and derived units to make larger or smaller units.

Prefix abbreviation	Factor
tera (T)	10^{12}
giga (G)	10^{9}
mega (M)	10^{6}
kilo (k)	10^{3}
deci (d)	10^{-1}
centi (cm)	10^{-2}
milli (m)	10^{-3}
micro (μ)	10^{-6}
nano (n)	10^{-9}
pico (p)	10^{-12}

Examples:

Derived Quantity	Equation	Unit Name	Symbol	Derived Unit
Force (*F*)	F = ma	Newton	N	kg m s^{-2}
Pressure (*p*)	$P = \frac{F}{A}$	Pascal	Pa	$\frac{\text{kg m s}^{-2}}{\text{m}^2} = \text{kg m}^{-1}\text{ s}^{-2}$
Energy (*E*)	E = mgh	Joule	J	(kg) (ms^{-2}) (m) = kg m^2s^{-2}
Power (*P*)	$P = \frac{\Delta E}{\Delta t}$	Watt	W	$\frac{\text{kg m}^2\text{ s}^{-2}}{s} = \text{kg m}^2\text{ s}^{-3}$
Frequency (*f*)	$f = \frac{1}{T}$, where T is period	Hertz	Hz	$\frac{1}{s} = s^{-1}$

HOMOGENEITY

- A physical equation is **homogeneous** if each of the terms, separated by mathematical operators has the same base units, on both sides of the equation.
- An equation which is not **homogeneous** is definitely wrong.
- All physically correct equations (physics equations), must be homogeneous.
- NOT ALL homogeneous equations are physically correct.

  Example: $KE = mv^2$. This equation is physically wrong but still homogeneous

  Example: $s = ut + \frac{1}{2}at^2$. This equation is homogeneous and physically right as $[s]=[ut]=\left[\frac{1}{2}at^2\right]$

ERRORS AND UNCERTAINTIES

- **Systematic errors** are errors of measurement in which measured quantities are displaced from the true value by fixed magnitude and in the same direction.
- Since a **systematic error** is a consistent error, it cannot be eliminated by taking the average of multiple readings. However, it can be eliminated if the source of the error is known.
- **Random errors** are errors of measurement in which measured quantities differ from mean value with different magnitudes and directions
- **Random errors** cannot be eliminated even if the source of error is known, as it is impossible to reproduce the exact same conditions for each measurement. However, random errors can be reduced by taking several observations of the same readings and then finding the mean or average.

Examples of random errors:

Error source	Examples	Corrections
Limitations of the observer	• Human reaction time when using a stop watch to measure the period of oscillation • Error of judgment, e.g. parallax error	• Take the time of 20 oscillations and find the mean period • Ensure the eye is properly aligned with the instrument when taking measurements.
Environmental conditions	• Fluctuation conditions e.g. pressure, temperature and current	• Take the average of repeated readings.
Intrinsic irregularity of quantity to be measured	• Small fluctuation in diameter of wire due to imperfect manufacture	• Measure the diameter at several points along the wire and take the average.

- Random errors reduce the **precision** of the measurements.
- The **precision** of an experiment is a measure of how reproducible the results are
- **Precision** is used to indicate the closeness with which the measurements agree with each other.

Examples of systematic errors:

Error source	Examples	Corrections
Instrumental error	• Zero error (pointer of instrument does not coincide with zero mark) • Incorrect calibration • End error (due to wear and tear of instrument)	• Always check for zero error and correct readings accordingly • Check instrument with standard instrument
Poor experimental technique	• Constant misalignment of ruler in measuring height	• Use a spirit level or plumbline to ensure rulers are vertical at all times
External factors	• Constant background radiation • Constant heat loss to surroundings	• Measure background count and subtract from reading • Conduct experiments with different variables and eliminate heat loss by simultaneous equations.

- Systematic errors reduce the **accuracy** of measurements
- **Accuracy** is a measure of the closeness of the measurements to the true or actual value

UNCERTAINTY

- Uncertainty in measurements is unavoidable.
- It estimates the range within which the answer is likely to lie.
- **Uncertainty** is expressed in the form of A ±Δ A, where ΔA is the uncertainty expressed to 1 s.f. and *A* is the measurement expressed to same number of d.p. as the uncertainty.

Uncertainty Calculations

1. Addition/ Subtraction: $A = B + C$ or $A = B - C \rightarrow \Delta A = \Delta B + \Delta C$
2. Division/ Multiplication: $A = \frac{B}{C}$ or $A = BC \rightarrow \frac{\Delta A}{A} = \frac{\Delta B}{B} + \frac{\Delta C}{C}$
3. Power: $A = B^n \rightarrow \frac{\Delta A}{A} = n\frac{\Delta B}{B}$
4. Complex Cases : Use First Principles, $\underbrace{\frac{\max + \min}{2}}_{\text{true value}} \pm \underbrace{\frac{\max - \min}{2}}_{\text{uncertainty}}$

SCALARS AND VECTORS

- **Scalar quantity** has only magnitude and no specified direction
- **Vector quantity** has both magnitude and a specified direction

Changes in velocity

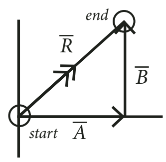
Example: John travels north then travels south. What is the velocity change?

Answer: Final velocity vector – Initial velocity vector

Vector diagrams

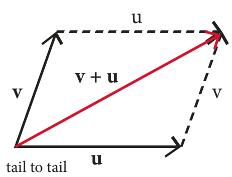
Head to Tail

- Draw the first vector. Then continue from the head of that vector to join the tail of the second vector. The resultant is from the start (tail) to the end (head).



Parallelogram Law

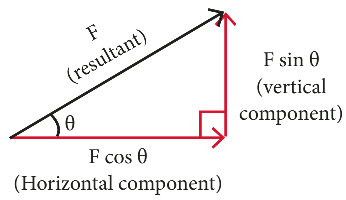
- Draw the two vectors tail to tail.
- Draw lines parallel to each vector to create a parallelogram. The tail of both vectors should start at the same point.



- Draw the resultant force joining 2 ends of the parallelogram as shown.

Vector Resolution

- Find the x and y components of the vectors.
- Add all x components. This gives the x component of the resultant vector.

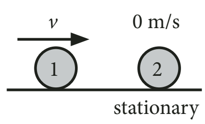


- Add all y components. This gives the y component of the resultant vector.
- Employ Pythagoras Theorem to get the magnitude of the resultant vector.
- The direction can be found out by trigonometry.

Relative velocity

1-Dimensional case

Example: Find the velocity of object 1 with respect to object 2.

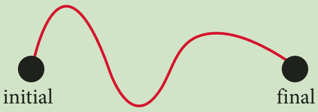
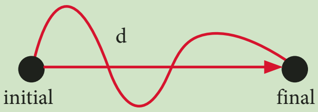


Answer: Velocity of 1 – velocity of 2
= v – 0
= v

CHAPTER 2 Kinematics

CONCEPT FIRST Your Physics Tuition Specialist

RECTILINEAR MOTION

Quantity (scalar/vector)	Definition
Distance, d **(scalar)**	The total length of the path travelled by a moving object, irrespective of the direction of motion. 
Displacement, s **(vector)**	The linear distance of the position of an object with reference to a given position (origin). Distance and direction must be specified 
Instantaneous speed, u **(scalar)**	Rate of change of distance, $u = \frac{dd}{dt}$
Average speed, <u> **(scalar)**	$\langle u \rangle = \frac{\text{total distance travelled}}{\text{total time taken}} = \frac{\Delta d}{\Delta t}$
Instantaneous velocity, v **(vector)**	Rate of change of displacement, $v = \frac{ds}{dt}$
Average velocity, <v> **(vector)**	$\langle v \rangle = \frac{\text{total displacement}}{\text{total time taken}} = \frac{\Delta s}{\Delta t}$
Instantaneous acceleration, a **(vector)**	Rate of change of velocity, $a = \frac{dv}{dt}$
Average acceleration, <a> **(vector)**	$\langle a \rangle = \frac{\text{change in velocity}}{\text{total time taken}} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$

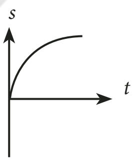
Sign conventions for Acceleration

V	a	effect	remarks
→	↠	Object speeds up & moves to the right	Positive acceleration
→	↞	Object slows down & moves to the right	Deceleration
←	↞	Object speeds up & moves to the left	Negative acceleration
←	↠	Object slows down & moves to the left	Negative deceleration

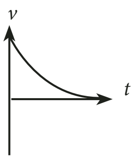
Graph description:

Scenario: Object is slowing down in the positive direction

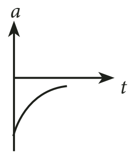
- *s* is positive
- *s* increases at a decreasing Rate in the positive direction
- Gradient of the graph gives v-t graph



- *v* is positive
- *v* decreases at a decreasing rate in the positive direction, till velocity reaches 0
- Area under the graph gives displacement, *s*
- Gradient of the graph gives acceleration, *a*



- *a* is negative
- Decreasing deceleration at decreasing rate in the positive direction
- Area under the graph gives velocity, *v*

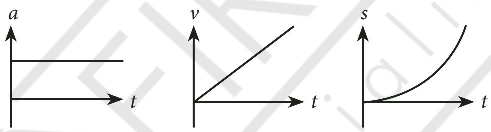


Graphs:

Free fall:

- A free-falling object is falling ONLY under the influence of gravity
- Does not encounter air resistance.

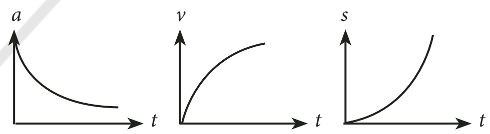
A ball is released from a top of a cliff, with no air resistance acting on it. Taking downwards as positive.



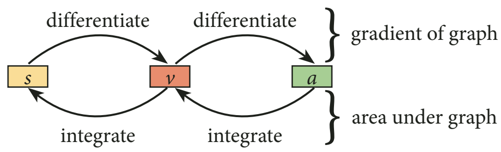
Object falling with resistance:

- In reality, air resistance on a falling object is taken into consideration.
- Air resistance opposes motion and tends to increase with velocity.
- The larger the velocity, the larger the opposing resistive force.
- Resultant acceleration is the combined free-fall acceleration and deceleration due to air resistance.

A ball is released from a top of a cliff, with air resistance acting on it. Taking downward as positive.



Relation between graphs:



PROJECTILE MOTION

Solving for Projectile Motion with no air resistance

Step 1: Define the positive direction for both horizontal (x) and vertical (y) components.

Step 2: Resolve the initial velocity, u, into its horizontal (x) and vertical (u) components.

Step 3: Fill up the table

x-direction : s_x, u_x, t	y-direction : s_y, u_y, v_y, a_y, t
s_x =	s_y =
u_x =	u_y =
v_x =	v_y =
a_x =	a_y =
t =	t =

* For maximum range angle of projection should be 45°

For x- direction: use only x components

For y- direction: use only y components

$v = u + at$	$v^2 = u^2 + 2as$	$s = ut + \frac{1}{2}at^2$	$s = \frac{1}{2}(v + u)t$

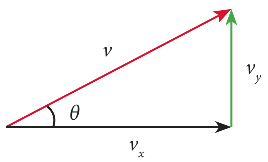
Conditions:

- The motion is 1 dimensional and linear.
- Acceleration is constant

Step 4: Apply the relevant equations of motion for x and y components.

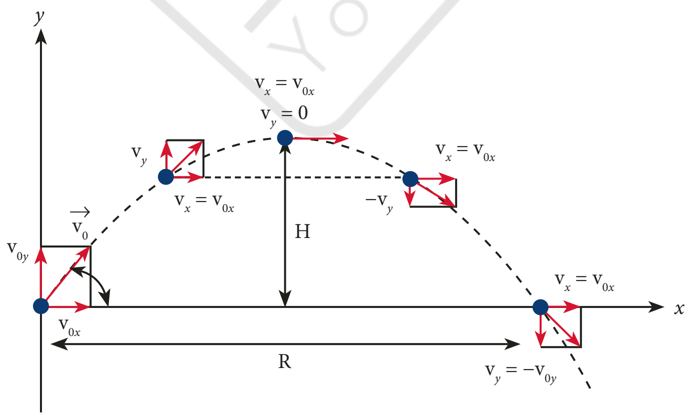
Step 5: To find the velocity vector at any time, we need the horizontal and vertical components, from which we can use Pythagoras' Theorem to get the magnitude and tangent to find the angle.

$$v = \sqrt{v_x^2 + v_x^2}, \quad \tan\theta = \frac{v_Y}{v_X}$$



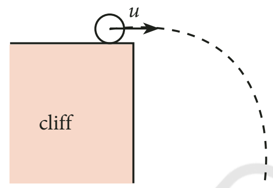
Assumptions for Projectile Motion

- **Acceleration** due to gravity, g, is **constant** (9.81 ms^{-2}) throughout entire motion
- g always directed downwards.
- Horizontal acceleration = 0, since g always points downwards.
- Negligible air resistance (unless otherwise stated in the question)
- Path of projectile without air resistance is always a symmetrical parabola

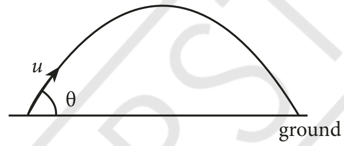


Various scenarios for projectile motion with no air resistance:

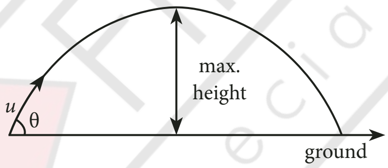
1. When object is projected horizontally ($u_y = 0$)



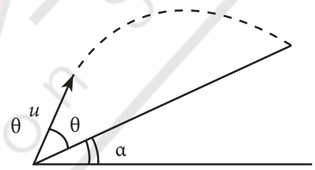
2. When object lands on the ground ($s_y = 0$)



3. At maximum height ($v_y = 0$)



4. Projectile motion up a slope



Projectile Motion with air resistance

- Smaller horizontal range
- Smaller maximum height
- Path asymmetrical about highest point
- Horizontal component of velocity no longer constant. It decreases with time
- Since | net acceleration upward | > | net acceleration downward |, | Δv of object going up | > | Δv of object going down |
- Object becomes slower at a faster rate on its way up
- Time taken to reach maximum point < time taken to reach the ground from the maximum point, i.e. $t_{up} < t_{down}$.
- This shows the asymmetry in the projection with air resistance.